REMARKS

In the Official Action mailed on **December 3, 2004** the Examiner reviewed Claims 1-18. Claims 1, 4, 6, 9, 11, 14, and 16 were objected to because of informalities. Claims 1-18 were rejected under 35 U.S.C. §112, second paragraph, as being indefinite for failing to particularly point out and distinctly claim the subject matter which applicant regards as the invention. Claims 1-18 were rejected under 35 U.S.C. §101 because the claimed invention is directed to non-statutory subject matter. Claims 1-2, 4-7, 9-12, and 14-15 were rejected under 35 U.S.C. §102(b) as being anticipated by Hansen (*Global Optimization using Interval Analysis*, hereinafter "Hansen").

Objections to the claims

Claims 1, 4, 6, 9, 11, 14, and 16 were objected to because of informalities.

Applicant has amended claims 1, 4, 6, 9, 11, 14, and 16 and canceled claim 3 to obviate the objections cited by the Examiner.

Rejections under 35 U.S.C. §112, second paragraph

Claims 1-18 were rejected as being indefinite for failing to particularly point out and distinctly claim the subject matter which applicant regards as the invention.

Applicant has amended independent claims 1, 6, 11, and 16 to clarify that the midpoints of the elements of **M** refers to the midpoints of the intervals comprising the elements of the matrix **M**. These amendments find support on page 10, line 22 to page 11, line 7 of the instant application.

Applicant respectfully points out that the hull of a system is well-known to practitioners with ordinary skill in the art. Applicant has appended a Power Point presentation concerning *Computational Geometry* from http://ranger.uta.edu/~gdas/student_slides/W11Presentation.ppt#1, which

provides an explanation of the hull and some computational processes related to the hull.

Applicant has amended claims 2, 7, 12, and 18 to clarify that the approximate center of matrix **A** is the approximate center of the interval elements of matrix **A**. These amendments find support on page 10, line 22 to page 11, line 7 of the instant application.

Applicant has amended claims 4, 8-9, 13-14, and 16 to clarify that r_i refers to elements of **r**. These amendments find support on page 11, lines 8-25 of the instant application.

Rejections under 35 U.S.C. §101

Claims 1-18 were rejected because the claimed invention is directed to non-statutory subject matter.

Applicant has amended independent claims 1, 6, 11, and 16 to clarify that the invention relates to solving global optimization problems, such as predicting weather, optimizing design of an aircraft engine, and solving non-linear systems. These amendments find support on page 10, lines 22-25 and on page 1, line 30 to page 2, line 3 of the instant application.

Rejections under 35 U.S.C. §102(b) and 35 U.S.C. §103(a)

Independent claims 1, 6, 11, and 16 were rejected as being anticipated by Hansen. Applicant respectfully points out that Hansen **fails to compute the hull** of the system (see Hansen, section 4.6, pages 29-31).

In contrast, the present invention **computes the hull** of the system (see page 12, lines 1-8 of the instant application). Computing the hull is beneficial because it provides a technique for bounding the solution set of the system. There is nothing within Hansen, either explicit or implicit, which suggests computing the hull of the system. Applicant respectfully points out that computing the hull of the system is a limitation of independent claims 1, 6, 11, and 16.

Hence, Applicant respectfully submits that independent claims 1, 6, 11, and 16 as presently amended are in condition for allowance. Applicant also submits that claims 2 and 4-5, which depend upon claim 1, claims 7-10, which depend upon claim 6, claims 12-15, which depend upon claim 11, and claims 17-18, which depend upon claim 16, are for the same reasons in condition for allowance and for reasons of the unique combinations recited in such claims.

CONCLUSION

It is submitted that the present application is presently in form for allowance. Such action is respectfully requested.

Respectfully submitted,

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Computational Geometry - Part II

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Convex Hull

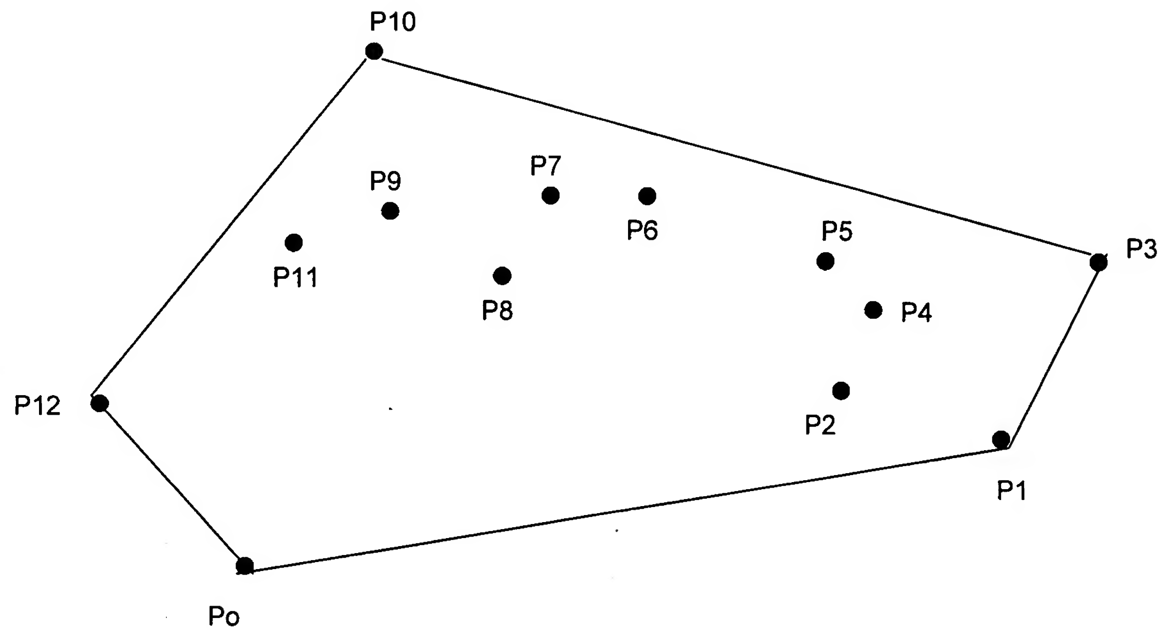
Definition:

The Convex Hull of a set Q of points is the smallest convex polygon P, for which each point in Q is either on the boundary of P or in its interior.

Intuition:

If there is a plane Q , consisting of nails sticking out from a board. Then the Convex Hull of Q can be thought of the shape formed by a tight rubber band that surrounds all the nails.

Example of Convex Hull



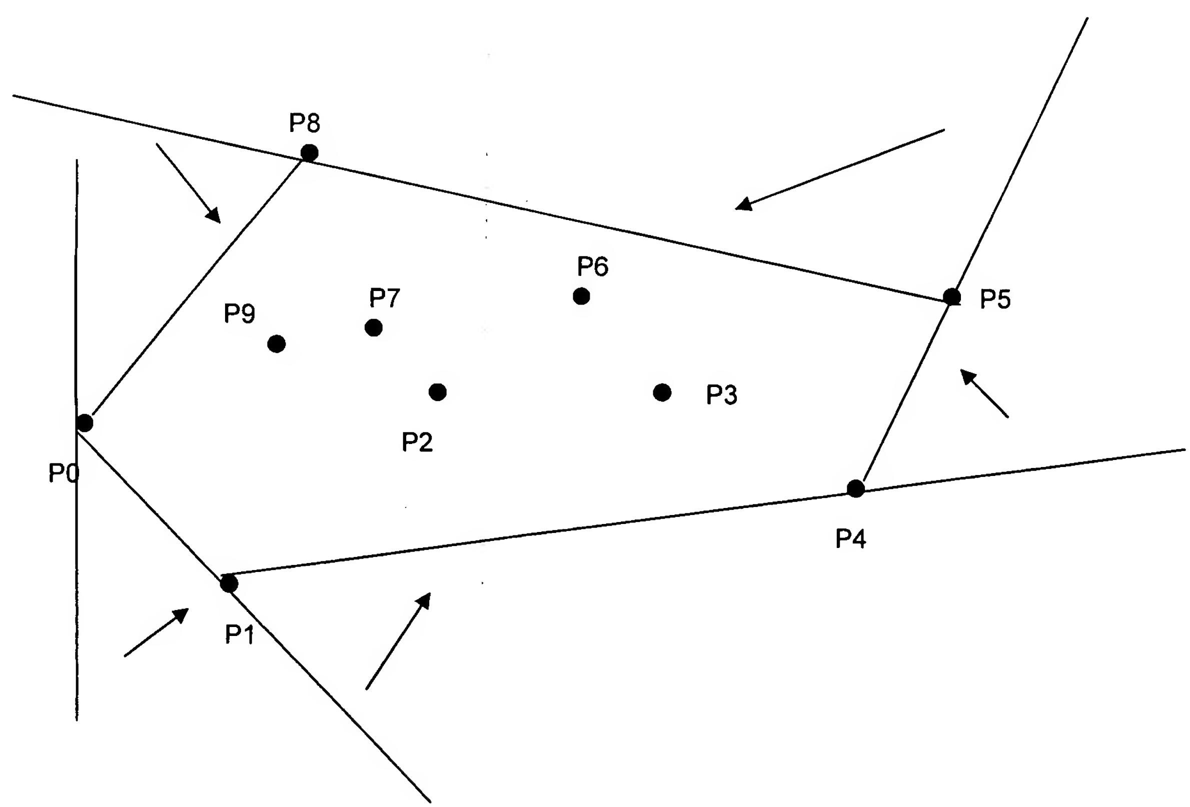
Jarvis's March

- Jarvis March computes the Convex Hull of a set Q of points by a technique called package wrapping or gift wrapping.

- Intuitively Jarvis March simulates a taut piece of paper around the set Q. To get the turning we take an "anchor" point then make a line with every other point and select the one with the least angle and keep on repeating.

- The algorithm runs in time O(nh) where h is the number of vertices

Example of Jarvis March



Jarvis March's algorithm starts from P0 ie the anchor point.

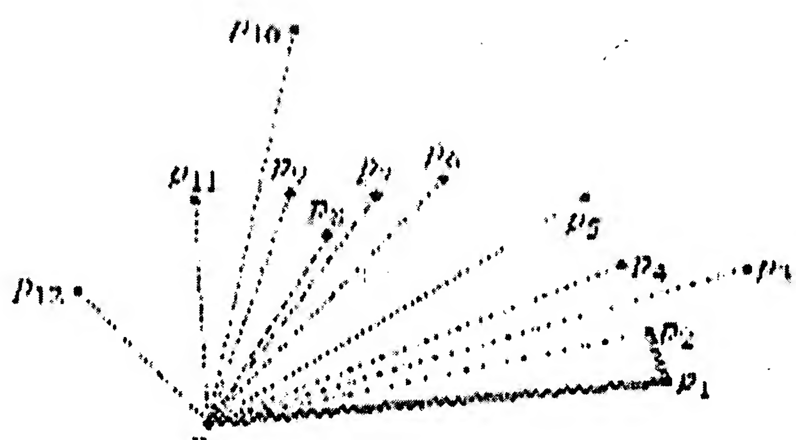
Then it wraps the next outer point and makes this as the new anchor point and repeats the procedure for the rest of the points.......

Till every point is wrapped inside.

Graham's Scan

- Graham Scan algorithm starts by taking the leftmost point.
- From this point calculate the angles to all other points
- Sort all the angles
- Start plotting to the next points
- Whenever it takes right turns it backtracks and re-joins those points that makes the shortest path.

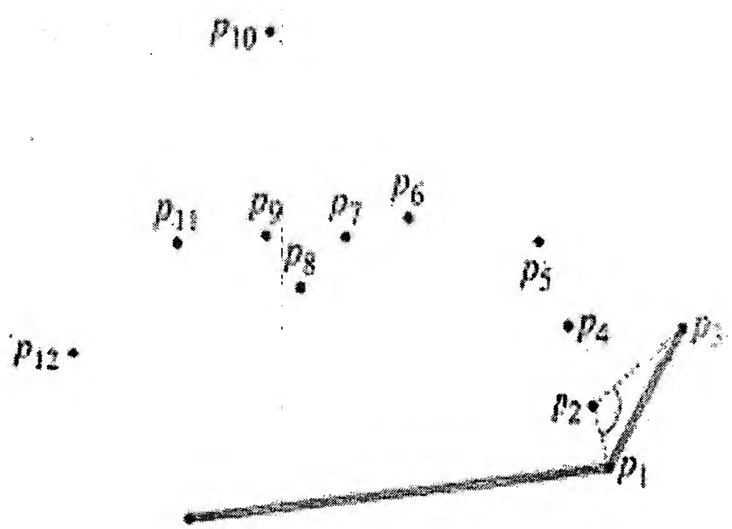
Example:



(a)

- As shown, Grahams starts from a point (p0) and calculates all the angles it makes to all the points and sorts the angles in an order

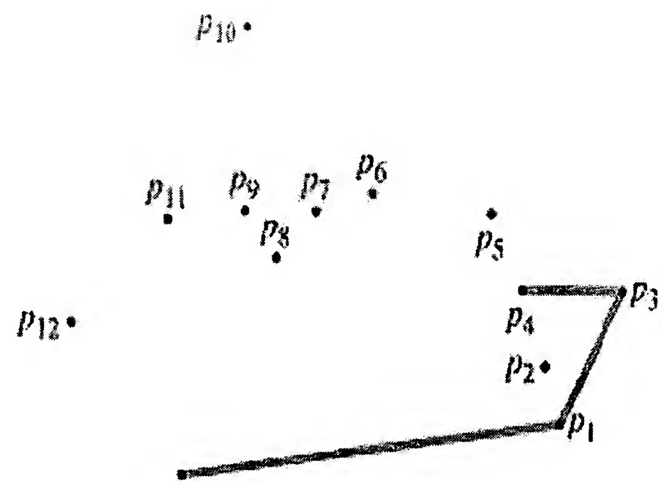
Example contd:



(b)

- It selects the point with the least angle and starts traversing (P0-P1).
- Then P1 to P2 & from P2 to P3 it realizes that it takes a right turn, so it backtracks and selects P1 – P3 directly, it being shorter

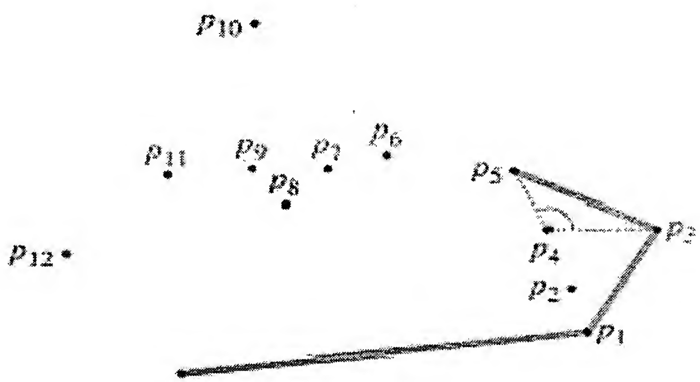
Example contd:



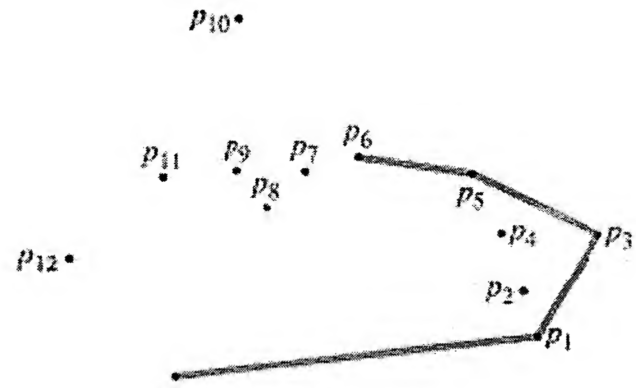
(c)

- The algorithm continues, based on the above mentioned conditions till it reaches back to the initial point. Hence forming the Convex Hull as shown:

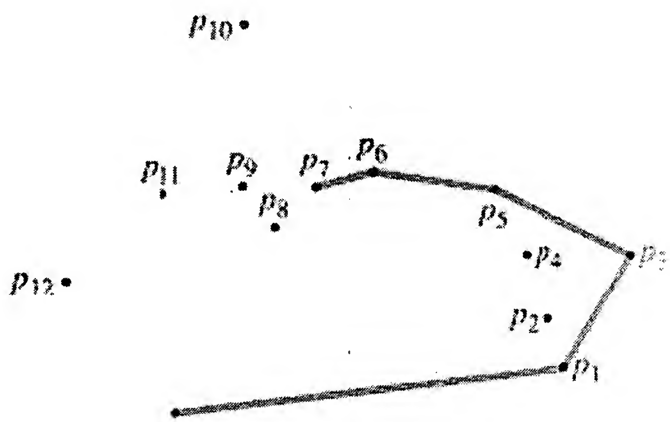
Example contd:



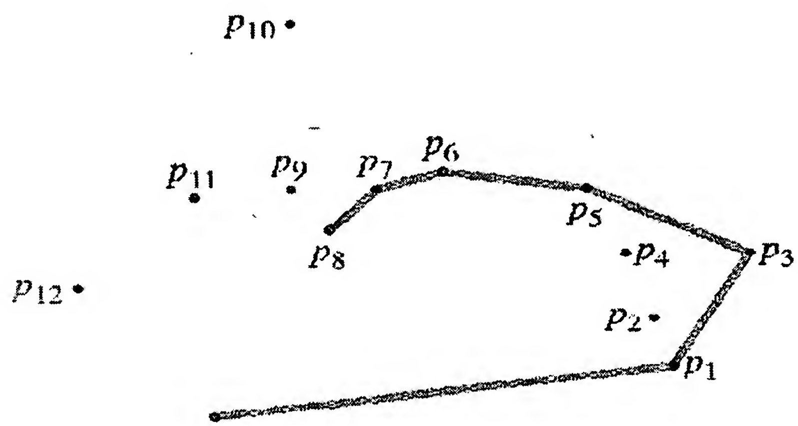
(d)



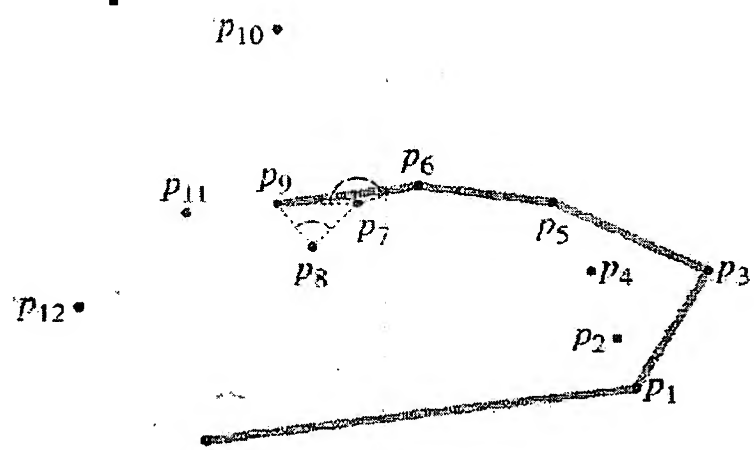
(c)



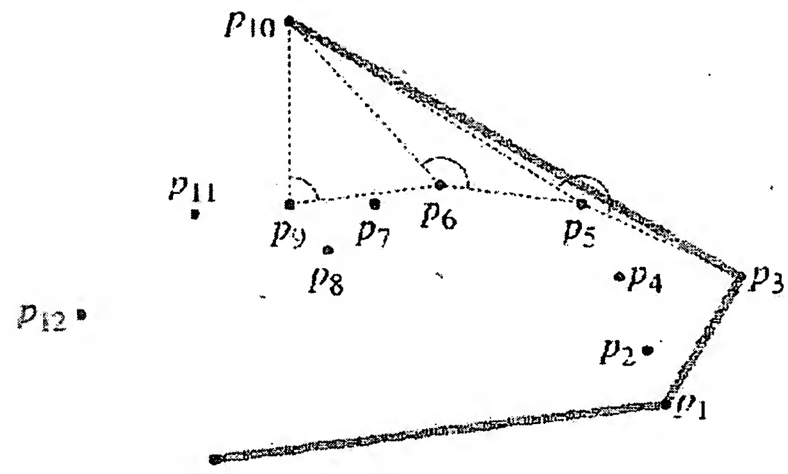
(f)



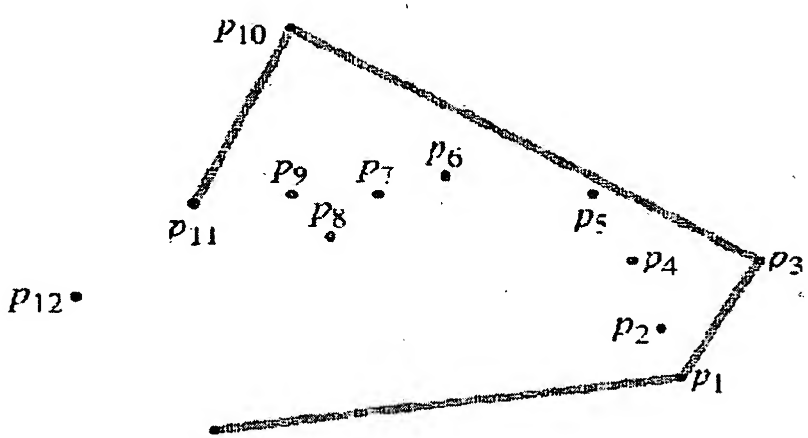
Example contd:



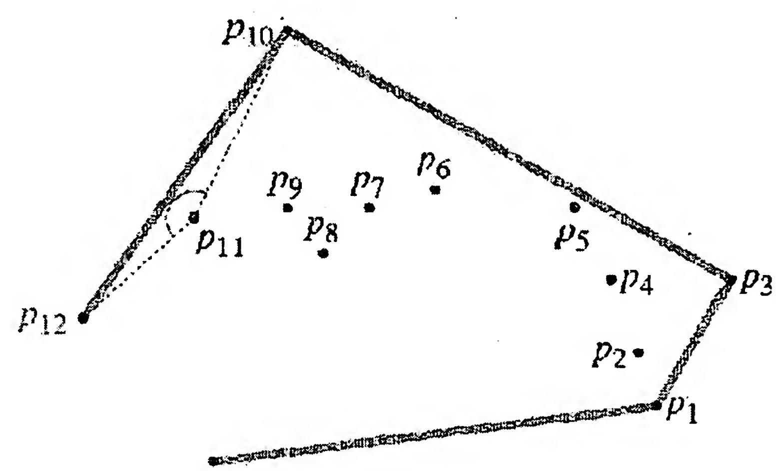
(h)



(i)

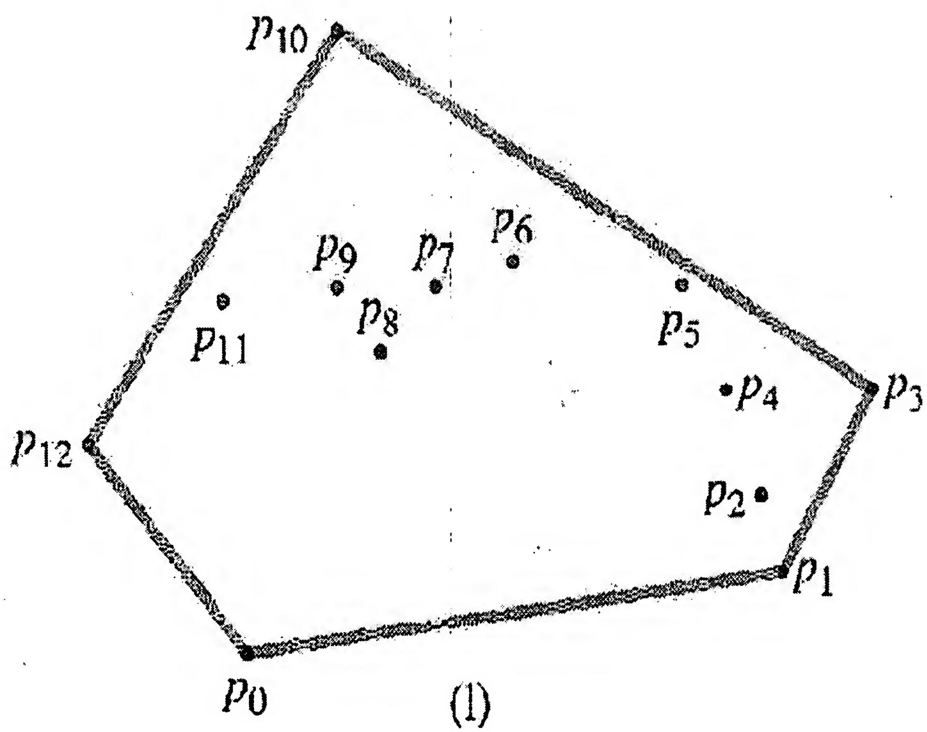


(j)



(k)

Example contd:



(1)

- Once the initial point is reached the algorithm self terminates, and the Convex Hull is formed.
- The algorithm takes O(nlgn) i.e the time it takes to sort the angles.

Strassen's algorithm

- The regular matrix multiplication algorithm takes $O(n^3)$.
- But strassen's algorithm uses dynamic programming to compute the matrix multiplication in $O(n^{2.81})$.

Strassen's explained:

- Suppose there are two matrices A and B giving the product matrix C.
- Then each matrix is divided into four sections as shown.

$$\overset{A}{\begin{bmatrix} a & b \\ c & d \end{bmatrix}} \overset{B}{\begin{bmatrix} e & f \\ g & h \end{bmatrix}} = \overset{C}{\begin{bmatrix} r & s \\ t & u \end{bmatrix}}$$

Strassen's contd:

C

$$\left[\begin{array}{c|c} r & s \\ \hline t & u \end{array}\right]$$

- The product matrix C can be defined by the following equations:

- r=ae+bg
- s=af+bh
- t=ce+dg
- u=cf+df

strassen’s contd:

- Each of the above entities in the previous equations take $T(n/2)$ time.
- Since we have eight such multiplications it takes $8T(n/2)+O(n^2)$, but strassen’s uses dynamic programming to reduce the number of multiplications to seven.
- Thus we have $7T(n/2)+O(n^2)$.

Strassen's explained:

- $s = af + bh$

  $= af - ah + ah + bh$

  $= \underbrace{a(f-h)}_{p1} + \underbrace{h(a+b)}_{p2}$

  $s = p1 + p2$

  p1 takes $T(n/2)+O(n^2)$

  p2 also takes $T(n/2)+O(n^2)$.

Strassen's contd:

- $t = ce + dg$

$$= ce - de + de + dg$$

$$= \underbrace{e(c-d)}_{p3} + \underbrace{d(e+g)}_{p4}$$

$t = p3 + p4$

p3 takes $T(n/2) + O(n^2)$

p4 also takes $T(n/2) + O(n^2)$.

Strassen's contd:

- r= ae+bg

  p5= (a+d) (e+h)
  = ae+ah+de+dh.

  p5 computes the inessential terms ah and de, which need to be cancelled some how. We can use p4 and p2 to cancel them, but two other inessential terms appear so hence we use p6 to cancel them.

  p5 +p4-p2= ae+dh+dg-bh
  And p6 is given as (b-d) (g+h)
  P6= bg+bh-dg-dh.

  Thus we obtain r= p5+p4-p2+p6
  = ae+bg. (hence achieved)

Strassen's contd:

- Similarly we obtain 'u' from p5 by using p1 and p3 to remove the inessential terms from p5.
- We have $p5+p1-p3 = ae+af-ce+dh$ and p7 is given as
  $p7= (a-e)(e+f)$
  $= ae+af-ce-cf$
- Leading us to $u = p5+p1-p3-p7$
  $= cf+dh$

Strassen's analysis

- Thus we computed only seven multiplications from p1-p7 using seven recursive calls.
- Thus $T(n)= 7T(n/2)+(n^2)$.
- Expanding:

  $n^2(\ldots\ldots\ldots(7/2^2)^3 + (7/2^2)^2 + 7/2^2 + 1)$

  we have logn such terms that can be represented by the formula $\frac{a.r^{n-1} - 1}{r - 1}$

Strassen's contd:

- Using the above formula we can simplify the series to $(7/4)^{\log n} + O(n^2)$

- This can also be represented as:

$= n^{\log 7/4} + O(n^2)$

$= n^{\log 7 - \log 4} + O(n^2)$

$= n^{\log 7 - 2} + O(n^2)$

$= n^{\log 7}/n^2 + O(n^2)$

$= n^2 \, (n^{\log 7}/n^2)$

$= n^{\log 7}$

Thus we get the complexity $O(n^{\log 7}) = O(n^{2.81})$.

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